1 Surface Flux

We have a bulk mesh and a surface mesh extracted from the bulk mesh. The aim is to compute the flux of a quantity through the surface mesh.

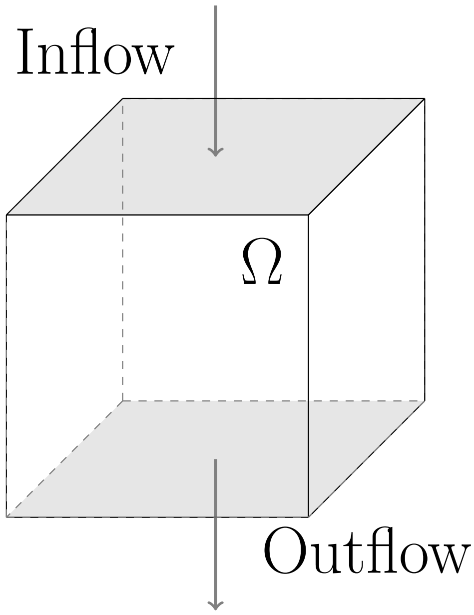

Figure 1.1: Domain Ω; Inflow and Outflow

For the computation of the bulk process the bulk mesh elements are transformed to the so called reference elements. In the following we denote the coordinates of the reference element as natural coordinates. In 3d we use for the first coordinate the letter r, for the second coordinate the letter s and for the third coordinate the letter t. In 2d the first coordinate is ξ and the second coordinate is η.

In OpenGeoSys the values of the quantity are computed in the gauss points of the bulk mesh elements using shape functions

$$N_i(r_j, s_j, t_j) = \delta_{ij}. \tag{1.1}$$

1.1 Surface

We want to approximate integral

$$\int_{\Gamma_e} \langle \boldsymbol{grad}\, p | n \rangle \, \mathrm{d}\Gamma$$

over a surface element by Gauss quadrature. Here the value of p at the place (ξ, η) has to be interpolated from the corresponding bulk mesh element. I.e., we need a mapping from (ξ, η) to (r, s, t) in order to access the value of p.

1.2 Triangle to Boundary Element Mappings

see fig. 1.2 - 1.2

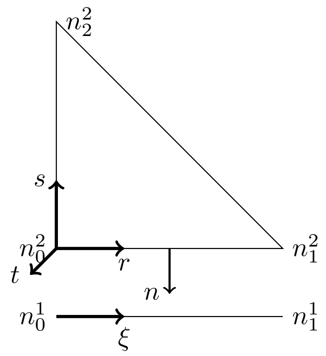

Figure 1.2: Mapping for boundary 0: $r = \xi$, $s = 0$, $t = 0$

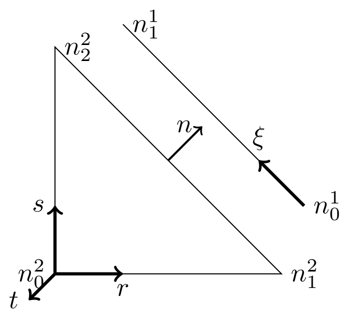

Figure 1.3: Mapping for boundary 1: $r = 1 - \xi$, $s = \xi$, $t = 0$

1.3 Hexahedron to Face Element Mappings

see fig. 1.5 - 1.10

1.4 Prism to Face Element Mappings

see fig. 1.11 - 1.4

1.5 Tetrahedron to Face Element Mappings

see fig. 1.16 - 1.19

1.6 Pyramid to Face Element Mappings

see fig. 1.20 - 1.24

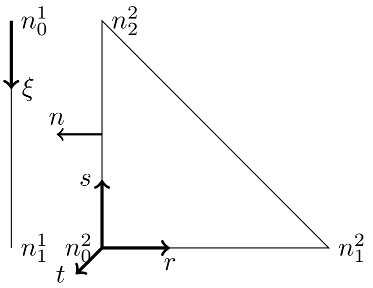

Figure 1.4: Mapping for boundary 2: $r = 0$, $s = 1 - \xi$, $t = 0$

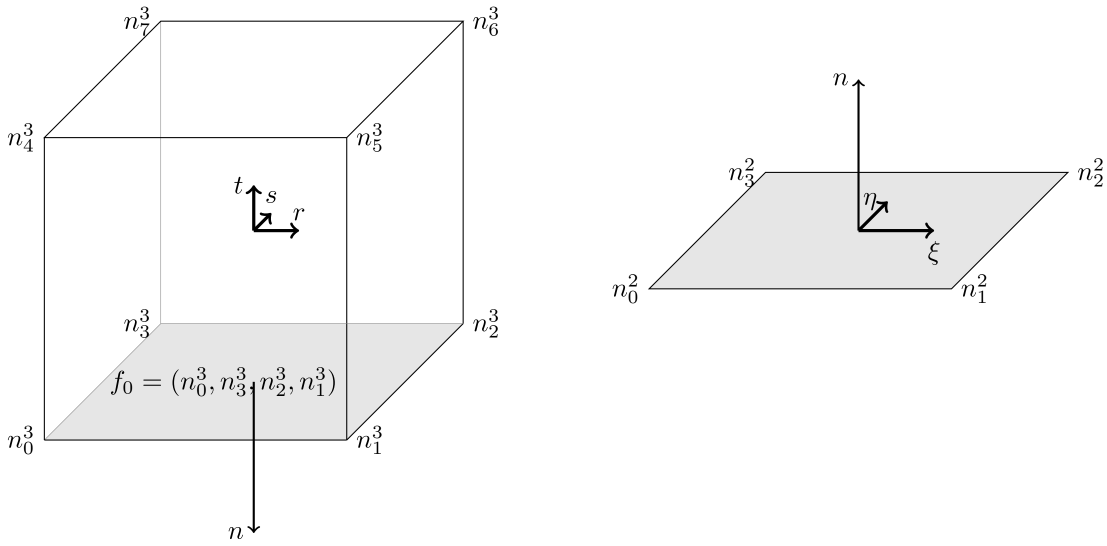

Figure 1.5: $r = \eta$, $s = \xi$, $t = -1$

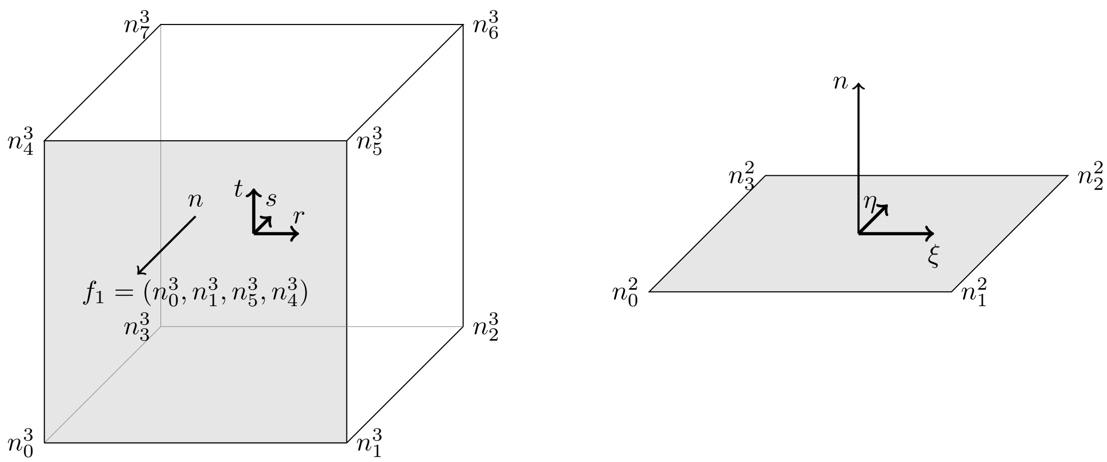

Figure 1.6: $r = \xi$, $s = -1$, $t = \eta$

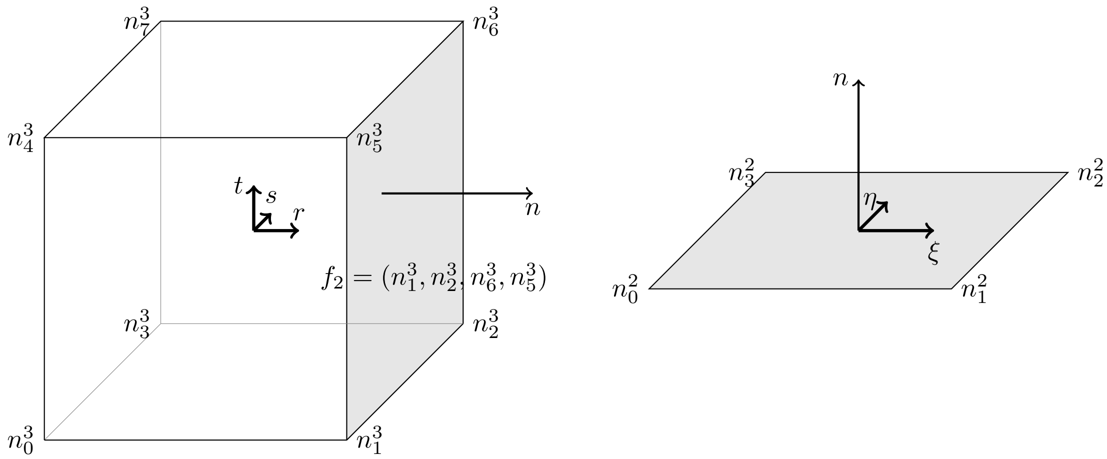

Figure 1.7: $r = 1$, $s = \xi$, $t = \eta$

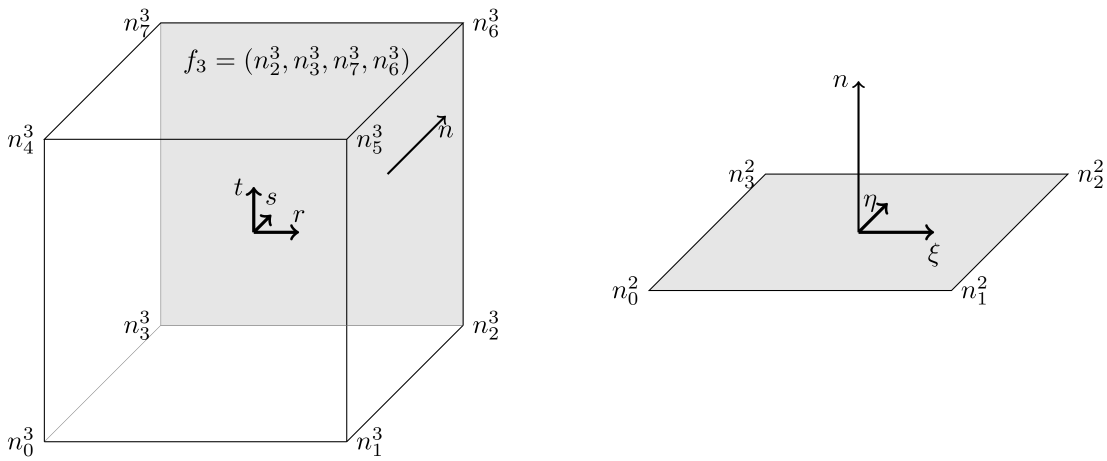

Figure 1.8: $r = -\xi$, $s = -1$, $t = \eta$

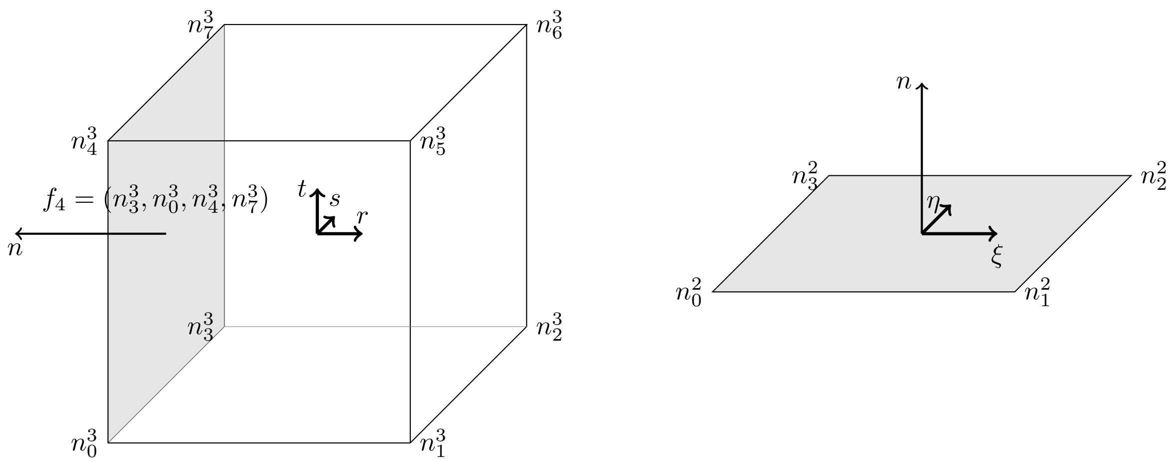

Figure 1.9: $r = -1$, $s = -\xi$, $t = -\eta$

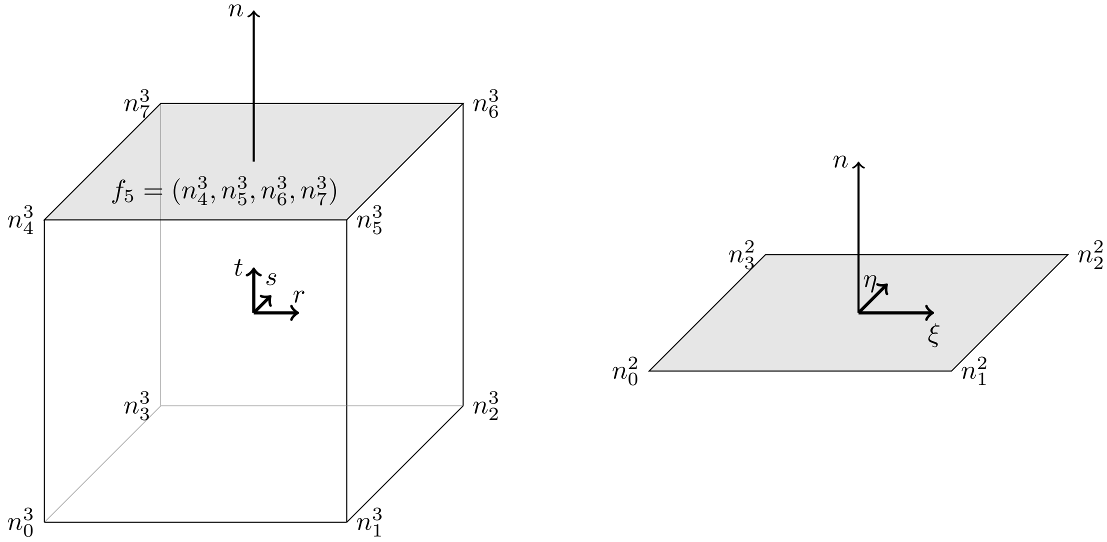

Figure 1.10: $r = \xi$, $s = \eta$, $t = 1$

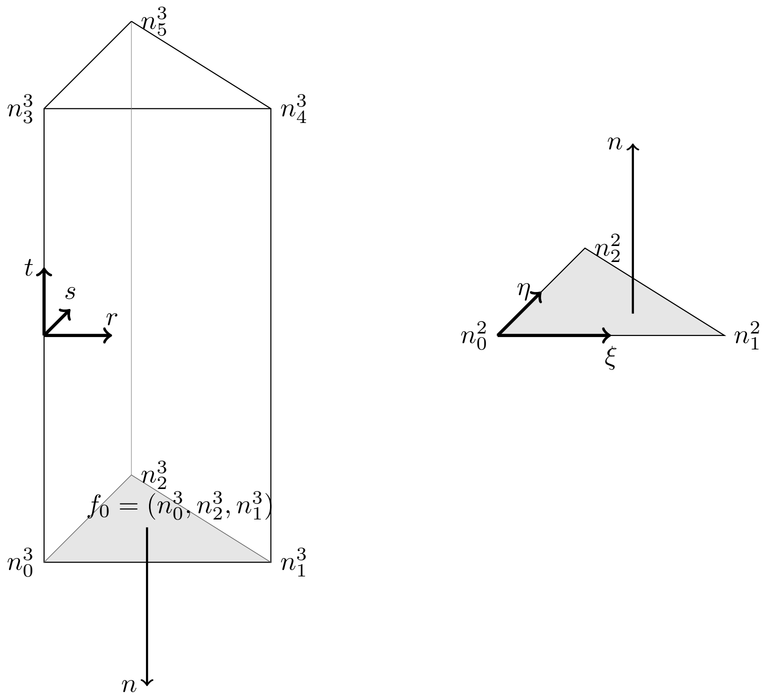

Figure 1.11: $r = \eta$, $s = \xi$, $t = -1$

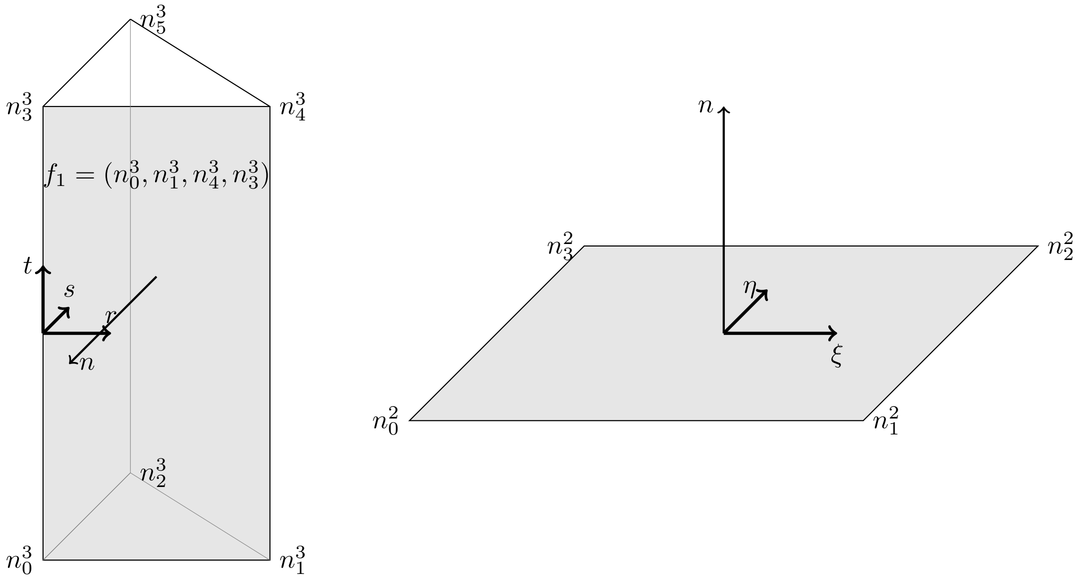

Figure 1.12: $r = \frac{\xi}{2} + \frac{1}{2}$, $s = 0$, $t = \eta$

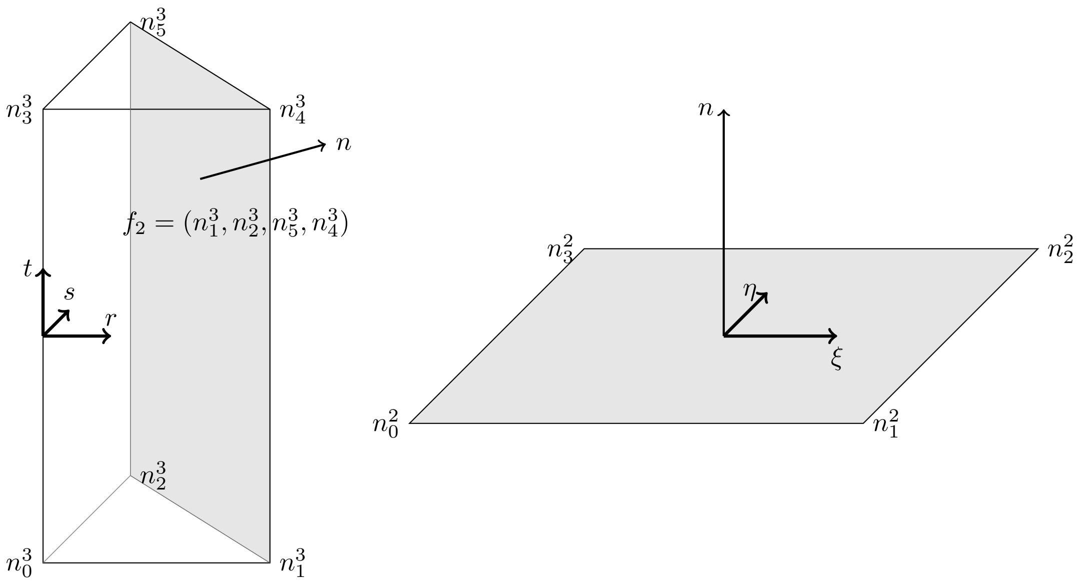

Figure 1.13: $r = \frac{1}{2} - \frac{\xi}{2}$, $s = \frac{1}{2} + \frac{\xi}{2}$, $t = \eta$

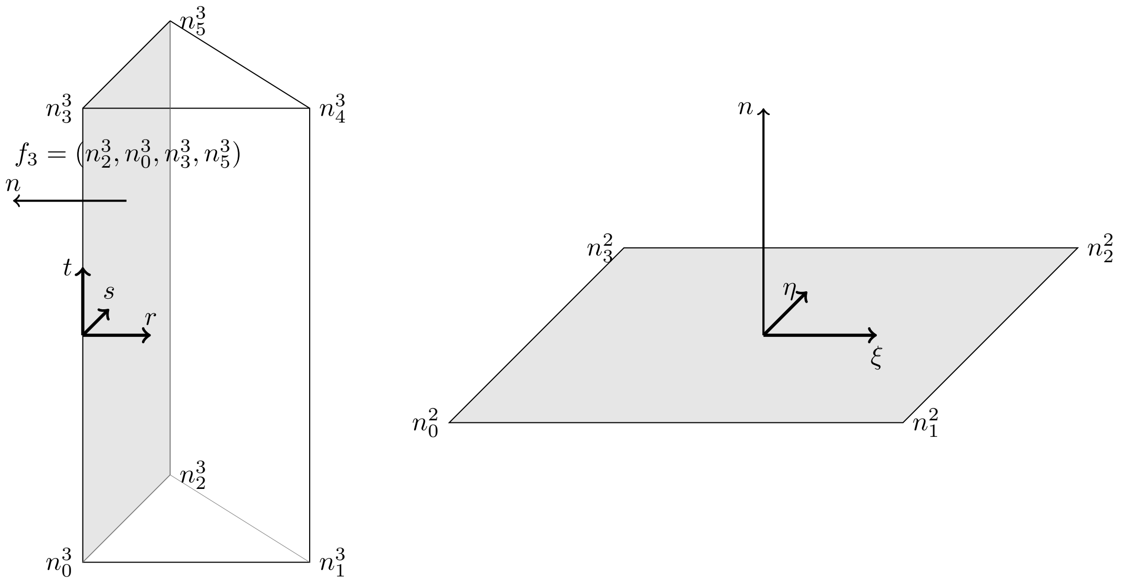

Figure 1.14: $r = 0$, $s = -\frac{\xi}{2} + \frac{1}{2}$, $t = \eta$

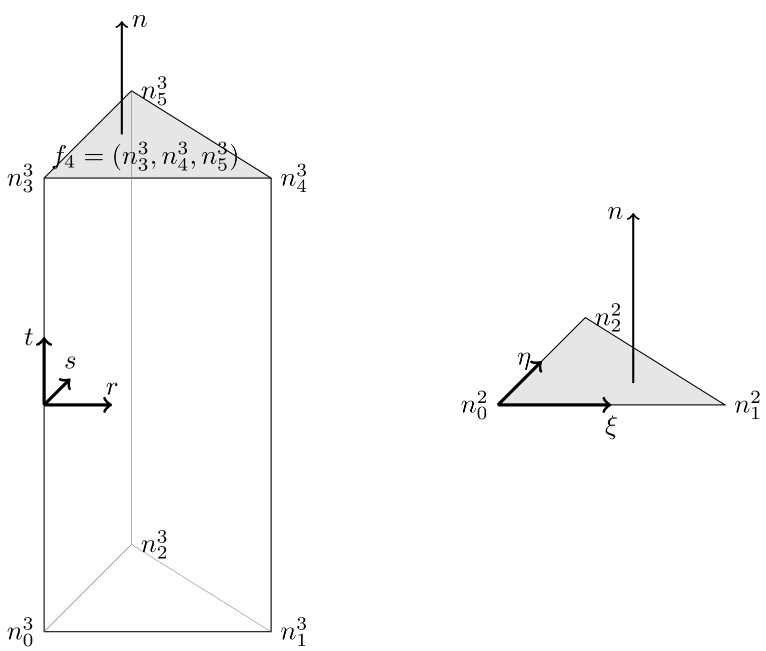

Figure 1.15: $r = \xi$, $s = \eta$, $t = 1$

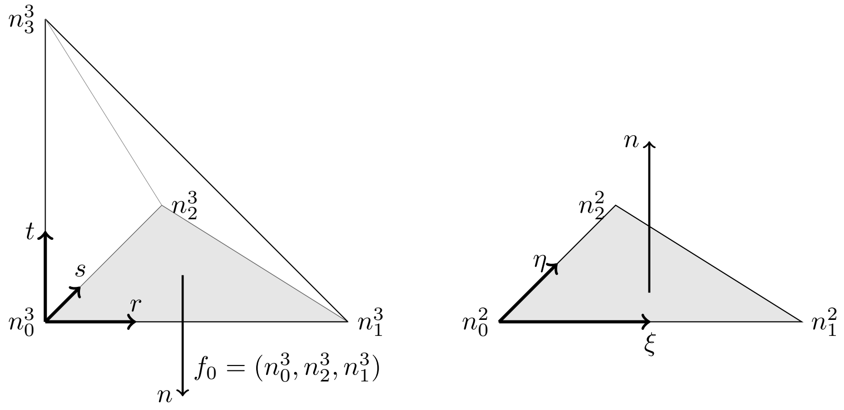

Figure 1.16: Mapping for $f_0 = (n_0^3, n_2^3, n_1^3)$: $n_0^2 \mapsto n_0^3$, $n_1^2 \mapsto n_2^3$, $n_2^2 \mapsto n_1^3 \Rightarrow r = \eta$, $s = \xi$, $t = 0$

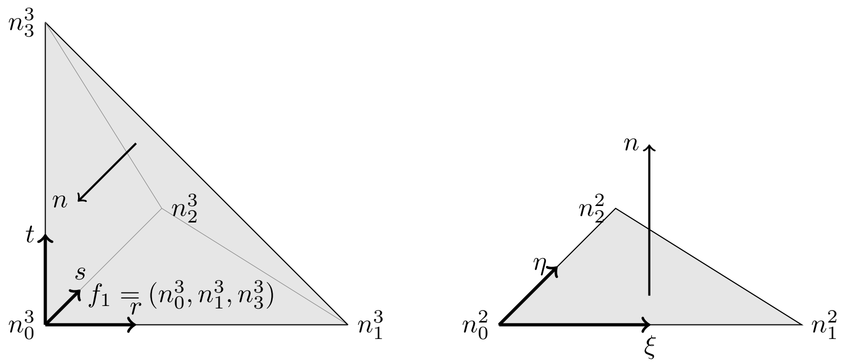

Figure 1.17: Mapping for $f_1 = (n_0^3, n_1^3, n_3^3)$: $n_0^2 \mapsto n_0^3$, $n_1^2 \mapsto n_1^3$, $n_2^2 \mapsto n_3^3 \Rightarrow r = \xi$, $s = 0$, $t = \eta$

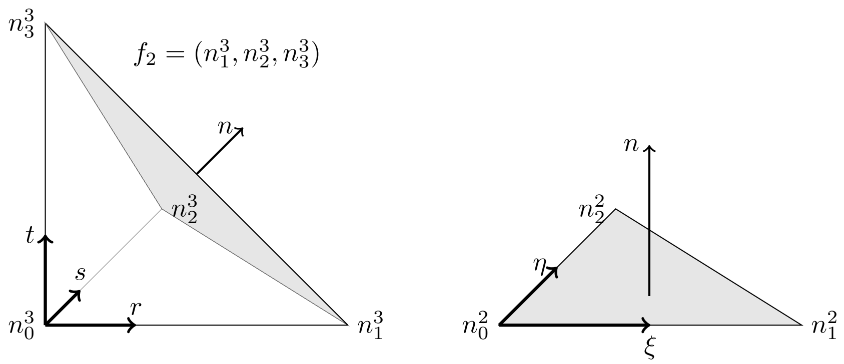

Figure 1.18: Mapping for $f_2 = (n_1^3, n_2^3, n_3^3)$: $n_0^2 \mapsto n_1^3$, $n_1^2 \mapsto n_2^3$, $n_2^2 \mapsto n_3^3 \Rightarrow r = 1 - \xi - \eta$, $s = \xi$, $t = \eta$

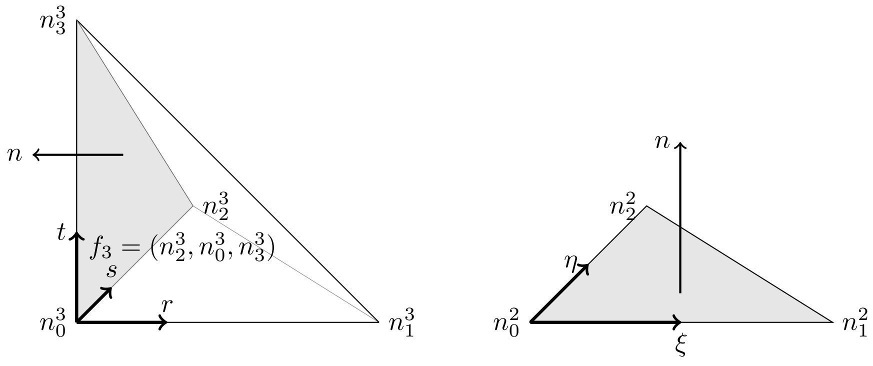

Figure 1.19: Mapping for $f_3 = (n_2^3, n_0^3, n_3^3)$: $n_0^2 \mapsto n_0^3$, $n_1^2 \mapsto n_3^3$, $n_2^2 \mapsto n_2^3 \Rightarrow r = 0$, $s = \eta$, $t = \xi$

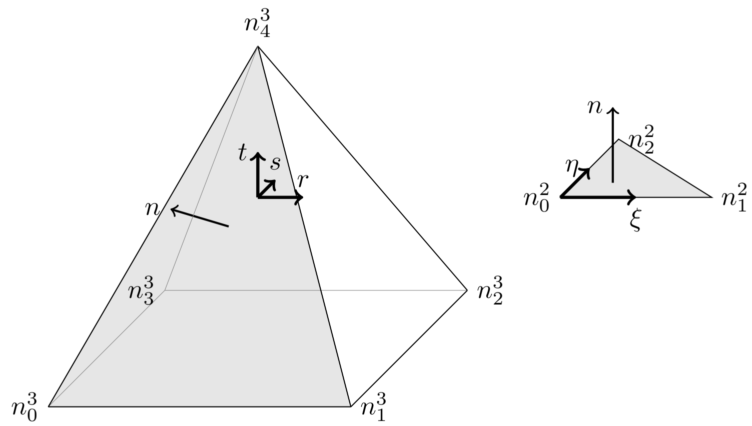

Figure 1.20: Mapping for $f_0 = (n_0^3, n_1^3, n_4^3)$: $n_0^3 \mapsto n_0^2$, $n_1^3 \mapsto n_1^2$, and $n_4^3 \mapsto n_2^2 \Rightarrow r = 2\xi - 1$, $s = -1$, $t = 2\eta - 1$

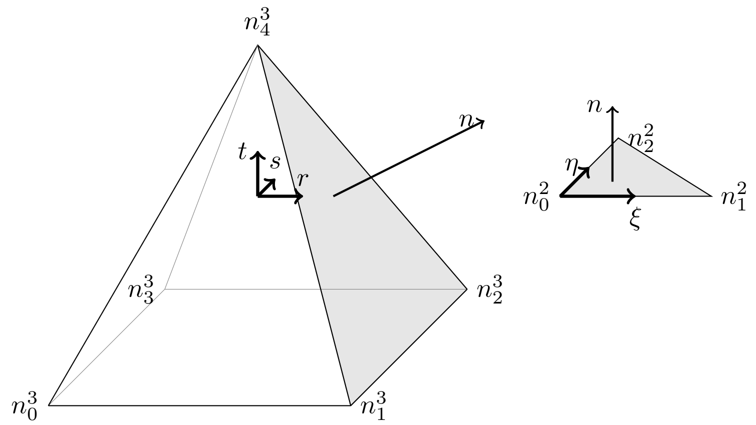

Figure 1.21: Mapping for $f_1 = (n_1^3, n_2^3, n_4^3)$: $n_1^3 \mapsto n_0^2$, $n_2^3 \mapsto n_1^2$, and $n_4^3 \mapsto n_2^2 \Rightarrow r = 1$, $s = 2\xi - 1$, $t = 2\eta - 1$

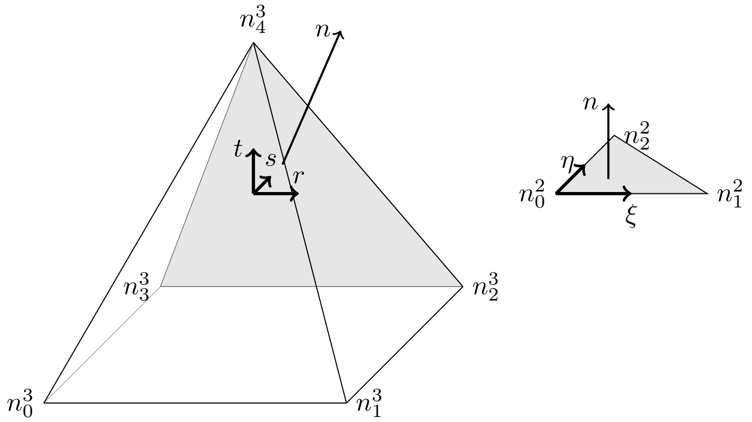

Figure 1.22: Mapping for $f_2 = (n_2^3, n_3^3, n_4^3)$: $n_2^3 \mapsto n_0^2$, $n_3^3 \mapsto n_1^2$, and $n_4^3 \mapsto n_2^2 \Rightarrow r = 1 - 2\xi$, $s = 1$, $t = 2\eta - 1$

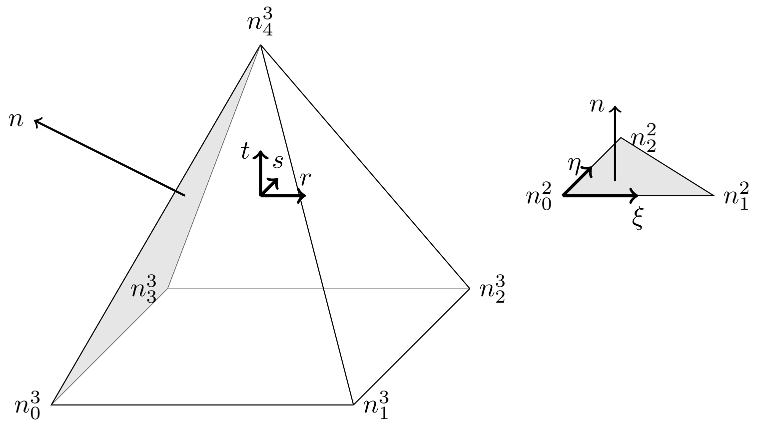

Figure 1.23: Mapping for $f_3 = (n_3^3, n_0^3, n_4^3)$: $n_3^3 \mapsto n_0^2$, $n_0^3 \mapsto n_2^2$, and $n_4^3 \mapsto n_1^2 \Rightarrow r = -1$, $s = 2\eta - 1$, $t = 2\xi - 1$

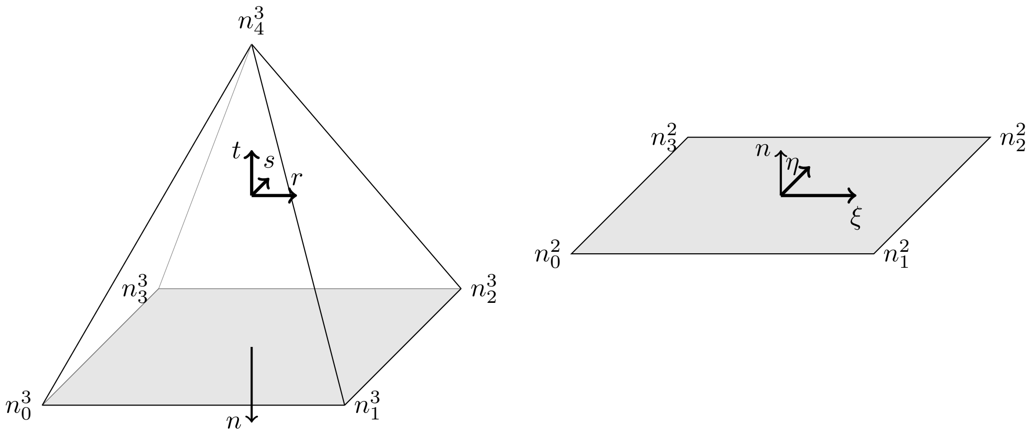

Figure 1.24: Mapping for$f_4 = (n_0^3, n_3^3, n_2^3, n_1^3)$: $n_0^3 \mapsto n_1^2$, $n_3^3 \mapsto n_2^2$, $n_2^3 \mapsto n_3^2$, and $n_1^3 \mapsto n_0^2 \Rightarrow r = -\xi$, $s = \eta$, $t = -1$