Small deformation including a fracture

This document describes small deformation of an elastic body with an embedded fracture. Propagation of the fracture is not considered.

1. Governing equations

Ignoring the gravity effect, one can formulate small deformation of an elastic solid as

$$\nabla \cdot \boldsymbol{\sigma} = 0 \quad \text{on } \Omega \tag{1.1}$$

$$\boldsymbol{\sigma} = \mathbf{D}\,\boldsymbol{\epsilon} \tag{1.2}$$

$$\boldsymbol{\epsilon} = \frac{1}{2}\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) \tag{1.3}$$

with boundary conditions

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_D, \tag{1.4}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}} \quad \text{on } \Gamma_N \tag{1.5}$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{D}$ is the fourth order elastic material tensor, and $\boldsymbol{\epsilon}$ is the strain vector, $\mathbf{u}$ is the solid displacement vector, $\bar{\mathbf{u}}$ is the prescribed displacement, and $\bar{\mathbf{t}}$ is the traction force vector.

Consider now the solid includes a fracture located on Γ_d within the domain. Sub-domains partitioned by the fracture are denoted as Ω^+ and Ω^-. The relative displacement vector between the fracture surfaces can be given as

$$[[\boldsymbol{u}]] = \boldsymbol{u}^+ - \boldsymbol{u}^- \quad \text{on } \Gamma_d \tag{1.6}$$

which can also be expressed in the local coordinates along the fracture as

$$\boldsymbol{w} = \boldsymbol{R}_d[[\boldsymbol{u}]] \tag{1.7}$$

with the rotation matrix $\mathbf{R}_d$. The fracture mean aperture b varies depending on the normal component of the relative displacement,

$$b = b_0 + w_n \geq 0 \tag{1.8}$$

where b_0 is the initial mean aperture. Forces acting on the upper and lower fracture surface are

$$\mathbf{t}_d^+ = \boldsymbol{\sigma} \cdot \mathbf{n}_d^+ \quad \text{on } \Gamma_d^+ \tag{1.9}$$

$$\mathbf{t}_d^- = \boldsymbol{\sigma} \cdot (-\mathbf{n}_d^+) \quad \text{on } \Gamma_d^- \tag{1.10}$$

with the normal vector $\mathbf{n}_d^+$ which is perpendicular to the fracture plane and is pointing to the subdomain Ω^+. If the fracture is totally open and has no contact, the above equilibrium equation has to satisfy the following condition

$$\mathbf{t}_d^+ = \mathbf{t}_d^- = 0. \tag{1.11}$$

On the other hand, if the fracture surfaces are in contact, one has to consider the continuity condition across the interface

$$\mathbf{t}_d^+ = -\mathbf{t}_d^- = \boldsymbol{K}\boldsymbol{w} \quad \text{on } \Gamma_d \tag{1.12}$$

where $\mathbf{K}$ is the fracture constitutive matrix.

2. Example

This example simulates small deformation of a rock sample having a pre-existing failure plane intersecting its core (Figure 1.1, left). The joint is inclined at angle $\beta = 50°$. The rock block has a height of 0.1 m and width of 0.05 m. Young's modulus is 170 MPa and Poisson's ratio is 0.22. The joint normal and shear stiffness are 1000 GPa/m and 1 GPa/m, respectively. The bottom of the rock is fixed vertically (roller boundary) and a uniaxial load of 7 kPa is applied to the top of the sample.

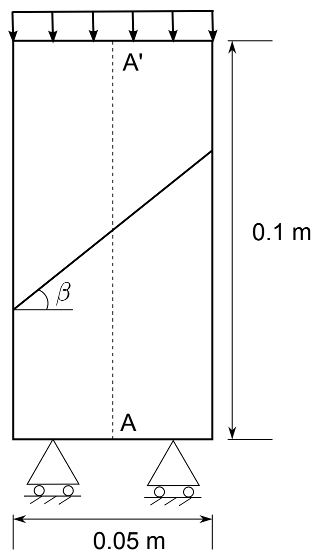

Figure 1.1: Problem definition with an inclined fracture

Simulation results are presented in Figure 1.2. As shown in the figure, the upper part of the domain slides to the left because of the inclined joint. The results agree well with the analytical solution.

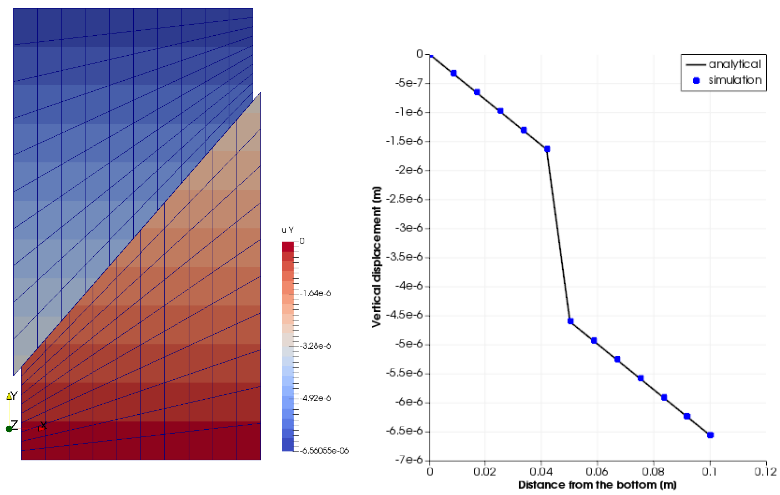

Figure 1.2: Deformation result (left) and profiles of vertical displacement at $x = 0.2$ m (right)