Benchmark for the failure index dependent permeability model

Wenqing Wang, Dmitri Naumov, Thomas Nagel, Olaf Kolditz

June 26, 2020

1 Theoretical Background

This implementation report presents the theoretical basis of the failure index dependent permeability model for the excavation damaged zone (EDZ) by taking into account HM response in rock due to excavation [2] as well as the implementation in OGS-6. The type of that permeability model in OGS-6 is `PermeabilityMohrCoulombFailureIndexModel`.

The failure index dependent permeability model according to [2] is defined as

$$\mathbf{k} = \mathbf{k}_0 + H(f-1)\, k_\mathsf{r}\, \mathrm{e}^{bf}\, \mathbf{I} \tag{1}$$

where $\mathbf{k}_0$ is the intrinsic permeability of the undamaged material, H is the Heaviside step function, f is the failure index, k_r is a reference permeability, b is a fitting parameter. k_r and b can be calibrated by experimental data.

The failure index f is calculated from the Mohr-Coulomb failure criterion comparing an acting shear stress. With the conventional mechanics notations, which mean that tensile stress is positive, the Mohr-Coulomb failure criterion [1] takes the form

$$\tau_\mathsf{f}(\sigma) = c - \sigma \tan\phi \tag{2}$$

with τ the shear strength, c the cohesion, σ the normal stress, and ϕ the internal friction angle. We further introduce the maximum shear stress $\tau_\mathsf{m} = (\sigma_3 - \sigma_1)/2$ and the mean stress $\sigma_\mathsf{m} = (\sigma_1 + \sigma_3)/2$, where σ_1 and σ_3 are the minimum and maximum shear stress, respectively. A second criterion is implemented similar to a tension cut-off. Let $\sigma_\mathsf{m}^\mathsf{max} \in (0, c/\tan\phi)$ be a limit value related (but not equivalent) to tensile strength of the material.

Then, the failure index is determined by

$$f = \begin{cases} \dfrac{|\tau_\mathsf{m}|}{\cos(\phi)\tau_\mathsf{f}(\sigma_\mathsf{m})} & \text{if } \sigma_\mathsf{m} \leq \sigma_\mathsf{m}^\mathsf{max} \\ \max\left\{\dfrac{|\tau_\mathsf{m}|}{\cos(\phi)\tau_\mathsf{f}(\sigma_\mathsf{m})}, \dfrac{\sigma_\mathsf{m}}{\sigma_\mathsf{m}^\mathsf{max}}\right\} & \text{if } \sigma_\mathsf{m} > \sigma_\mathsf{m}^\mathsf{max} \end{cases} \tag{3}$$

The computed permeability components are restricted with an upper bound, i.e. $\mathbf{k} := k_{ij} < k_\mathsf{max}$.

The material properties for the test example are given in Table 1. The parameters of the EDZ perme-

Table 1: Material properties

Property	Value	Unit
Fluid		
Density	1000	kg/m^3
Fluid viscosity	10^{-3}	Pa s
Solid		
Density	2650	kg/m^3
Porous medium		
Porosity	0.15	$-$
Intrinsic permeability	the EDZ model	m^2
Elasticity		
Young's modulus	$6 \cdot 10^9$	Pa
Poisson's ratio	0.3	-
Biot's coefficient	0.6	-

ability with the Mohr Coulumb failure index are

$$\begin{aligned}
&\mathbf{k}_0 = \{10^{-20}\}\,\text{m}^2,\\
&k_\text{r} = 10^{-19}\,\text{m}^2, b = 3.0,\ c = 1\,\text{MPa},\ \phi = 15^\circ,\\
&k_\text{max} = 10^{-6}\,\text{m}^2,\ \sigma_\text{m}^\text{max} = 0.8\frac{c}{\tan\phi} = 2.985640646055102e6\,\text{MPa}
\end{aligned} \tag{4}$$

This geometry of this example is a square of [0,50]×[-25,25] m^2 with a half circle hole with a radius of 2.3 m and a center at (0,0).

The initial pore pressure is 4.7 MPa. The initial displacement and stress components are all zero.

Table 2: Boundary

Boundary	Mass balance equation	Momentum balance equation
Left	No flux	$u_x = 0, \tau_y = 0$
Right	No flux	$\tau_x = -15\,\text{MPa}, \tau_y = 0$
Bottom	No flux	$u_y = 0, \tau_x = 0$
Top	No flux	$\tau_x = 0, \tau_y = -12\,\text{MPa}$
Hole surface	$p = 0.1\,\text{MPa}$	$\tau_n = 0$

The computed permeability is shown in Fig. 1, in which one can see that the permeability near the hole is increased with a reasonable distribution pattern. This implies that the permeability model can describe the permeability change in EDZ.

The distributions of the horizontal stress, σ_{xx}, and the pore pressure are illustrated in Fig. 2.

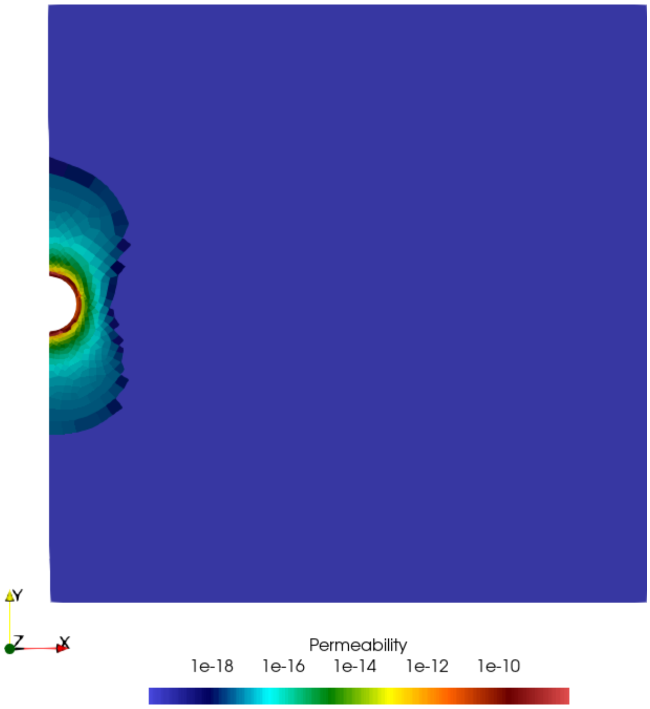

Figure 1: Calculated permeability distribution in [m^2].

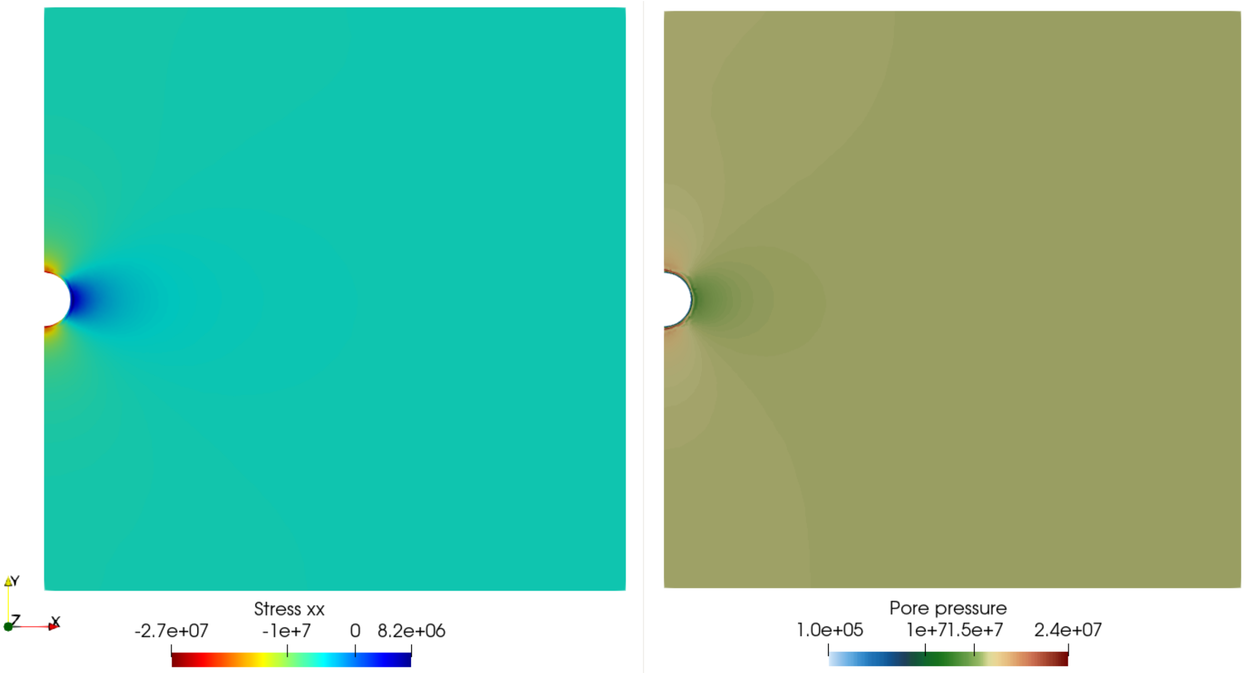

Figure 2: Calculated distributions of horizontal stress (left) and pore pressure (right), in [Pa] respectively

References

[1] J.F. Labuz and A. Zang. Mohr–Coulomb failure criterion. *Rock Mechanics and Rock Engineering*, 45(6):975–979, 2012.

[2] W.Q. Wang, H. Shao, Th. Nagel, and O. Kolditz. Analysis of coupled thermal-hydro-mechanical processes during small scale in-situ heater experiment in Callovo-Oxfordian clay rock introducing a failure-index permeability model. *International Journal of Rock Mechanics and Mining Sciences*, revised manuscript under review, 2020.